

A Yukawa coupling parameterization for type I+II seesaw formula and applications to lepton flavor violation and leptogenesis

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ABSTRACT: In the type I + II seesaw formula the mass matrix of light neutrinos m_ν receives contributions from the exchanges of both heavy Majorana neutrinos and $SU(2)_L$ -triplet Higgs bosons. We propose a new parameterization for the Dirac-type Yukawa coupling matrix of neutrinos in this case, which generalizes the well known Casas-Ibarra parameterization to type I + II seesaw and is useful when the triplet term in m_ν is known. Neutrino masses and mixing, lepton flavor violation in decays like $\mu \rightarrow e\gamma$ within mSUGRA models and leptogenesis can then be studied within this framework. We illustrate the usefulness of our new parameterization using a number of simple examples.

KEYWORDS: Neutrino Physics, Baryogenesis, Supersymmetry Phenomenology.

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1. Introduction

The seesaw mechanism [1, 2] provides a very natural and attractive explanation of the smallness of neutrino mass [3] as being due to exchanges of heavy particles. In the most commonly considered type I seesaw, these are heavy sterile (electroweak-singlet) Majorana neutrinos [1]; another well studied case is type II seesaw, where the small neutrino mass is generated by the induced vacuum expectation value (VEV) of an $SU(2)_L$ -triplet Higgs boson [2]. In both cases, the light neutrinos are Majorana particles with the effective mass matrix m_ν , which in the basis where the mass matrix of the charged leptons is diagonal and real, is diagonalized according to

$$m_\nu = U^* m_\nu^{\text{diag}} U^\dagger, \quad m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3). \quad (1.1)$$

Here U is the leptonic mixing matrix, which depends on three mixing angles, one Dirac-type and two Majorana-type CP-violating phases. Although both sterile neutrinos and Higgs triplets can be freely added to the standard model, they are most natural in its partially unified or grand unified extensions, such as left-right symmetric models or $SO(10)$ grand unified theories (GUTs), where both type I and type II contributions to the neutrino mass are typically present. In that case the neutrino mass matrix is a sum of two terms:¹

$$m_\nu = m_\nu^{\text{II}} + m_\nu^{\text{I}} = v_L f_L - \frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T. \quad (1.2)$$

Here the first term is the $SU(2)_L$ -triplet Higgs contribution with v_L the VEV of the triplet and f_L the triplet Yukawa coupling matrix. The triplet VEV $v_L \simeq \mu v_u^2 / M_\Delta^2$, where μ is

¹Note that sometimes in the literature the mechanism leading to the entire eq. (1.2) (rather than only to the triplet contributions to m_ν) is called type II seesaw.

the trilinear Higgs coupling, v_u is the VEV of the up-type Higgs doublet H_u , and M_Δ is the mass of the triplet. The second term in (1.2) is the conventional type I seesaw term, in which $v_u Y_D$ is the Dirac mass matrix m_D . Having in mind extensions of the standard model, we have written the Majorana mass matrix of heavy neutrinos M_R as $v_R f_R$, with f_R being the relevant coupling matrix. In particular, in left-right symmetric gauge theories f_R is the Yukawa coupling matrix of an $SU(2)_R$ -triplet Higgs and v_R is its VEV, which is related to the VEV of the $SU(2)_L$ -triplet via $v_L v_R \propto v_u^2$. Regardless of the variant of the seesaw mechanism and barring unnaturally small Yukawa couplings or strong cancellations, the typical mass scale of the neutrino mass generation (M_Δ or v_R or both) exceeds 10^9 GeV, which is way beyond the reach of direct experimental tests. Hence, the seesaw mechanism of neutrino mass generation can only be probed indirectly.

One way of indirectly probing the seesaw is provided by cosmology, where the observed baryon number of the universe can be generated through the baryogenesis via leptogenesis mechanism [4, 5]. Leptogenesis can work successfully within both type I [6, 7] and type II [8] seesaw scenarios,² as well as in the combined type I + II seesaw [9] (for earlier works see [10–12]), where the neutrino mass matrix is given by eq. (1.2). Another possibility of testing the seesaw is through lepton flavor violation (LFV) within supersymmetric theories, which has also been discussed in both type I [13] and type II [14–16] seesaw frameworks. Here LFV is induced by off-diagonal entries in the slepton mass matrices, which can be generated radiatively. In many well motivated scenarios, the size of these entries depends on the seesaw parameters. Leptogenesis and LFV depend on combinations of the Yukawa coupling matrices that are different from those entering into the seesaw formula (1.2), and this can be used — at least in principle — to reconstruct the seesaw parameters.

An important issue in these approaches is that the number of high energy parameters, i.e. of those contained in m_ν^{II} , m_D and M_R , exceeds the number of low energy parameters contained in m_ν , simply because the heavy degrees of freedom are integrated out at low energies. An exception is the case of type II dominance, when m_ν coincides (or approximately coincides) with m_ν^{II} . In the general case, however, and without a specific model at hand, one can only parameterize the unknown high energy quantities. Within the pure type I seesaw, one such parameterization, which proved to be especially useful and convenient, was suggested by Casas and Ibarra [17]. This is the parameterization of the Dirac-type Yukawa coupling matrix Y_D in which it is written as

$$v_u Y_D = i U^* \sqrt{m_\nu^{\text{diag}}} R \sqrt{M_R^{\text{diag}}} = i \sqrt{v_R} U^* \sqrt{m_\nu^{\text{diag}}} R \sqrt{f_R^{\text{diag}}}. \quad (1.3)$$

Here R is a complex orthogonal matrix that contains the parameters which are integrated out when m_ν is obtained, and which therefore cannot be determined from low energy neutrino data without additional input. Note that this parameterization is valid in the basis in which the charged lepton Yukawa coupling is real and diagonal. Many analyses of neutrino mixing, LFV and/or leptogenesis in the type I seesaw framework have been performed using this parameterization [18, 19]. However, to the best of our knowledge, no parameterization of this kind has been suggested for the general case of type I + II seesaw.

²In the case of pure type II seesaw more than one Higgs triplet is necessary for leptogenesis to work.

The purpose of the present paper is to generalize the Casas-Ibarra parameterization to the case of the combined type I + II seesaw, when the mass matrix of light neutrinos is given by eq. (1.2), and to demonstrate the usefulness of the proposed parameterization.

The paper is organized as follows. We summarize the main aspects of lepton flavor violation, leptogenesis and neutrino mixing within type I + II seesaw in section 2. Section 3 contains our central results. Here we introduce our parameterization of the Dirac-type Yukawa coupling matrix in the case of the combined type I + II seesaw. We also give simple examples on its usage, two of which are based on the approximate tri-bimaximality of neutrino mixing. We conclude in section 4.

2. Formalism

We will work in the basis in which the mass matrix of charged leptons is real and diagonal. The mass matrix of light neutrinos is then diagonalized according to eq. (1.1), with U the leptonic mixing matrix, for which we will use the parameterization

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix} P. \quad (2.1)$$

Here $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, δ is the Dirac-type CP-violating phase, and the Majorana phases α and β are contained in the matrix

$$P = \text{diag}(1, e^{-i\alpha}, e^{-i\beta}). \quad (2.2)$$

The analyses of neutrino experiments revealed the following best-fit values and 3σ ranges of the oscillation parameters [20]:

$$\begin{aligned} \Delta m_{\odot}^2 &\equiv m_2^2 - m_1^2 = (7.9_{-0.89}^{+1.1}) \cdot 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.31_{-0.05}^{+0.07}, \\ \Delta m_{\text{A}}^2 &\equiv |m_3^2 - m_1^2| = (2.6_{-0.6}^{+0.6}) \cdot 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{23} &= 0.47_{-0.15}^{+0.17}, \\ |U_{e3}|^2 &= 0_{-0.000}^{+0.040}. \end{aligned} \quad (2.3)$$

Depending on the sign of $m_3^2 - m_1^2$, the neutrino masses are normally or inversely ordered:

$$\begin{aligned} \text{normal: } m_3 > m_2 > m_1 &\quad \text{with} \quad m_2 = \sqrt{m_1^2 + \Delta m_{\odot}^2}; \quad m_3 = \sqrt{m_1^2 + \Delta m_{\text{A}}^2}, \\ \text{inverted: } m_2 > m_1 > m_3 &\quad \text{with} \quad m_2 = \sqrt{m_3^2 + \Delta m_{\odot}^2 + \Delta m_{\text{A}}^2}; \quad m_1 = \sqrt{m_3^2 + \Delta m_{\text{A}}^2}. \end{aligned} \quad (2.4)$$

The overall scale of neutrino masses is not known, except for the upper limit of order 1 eV coming from direct mass search experiments and cosmology.

The type I seesaw mechanism [1] corresponds to the situation when the light neutrino masses are induced by their coupling with heavy Majorana neutrinos. Introducing the

Dirac mass matrix $m_D = v_u Y_D$ with $v_u = v \sin \beta$ being the VEV of H_u , and the Majorana mass matrix $M_R = v_R f_R$ for heavy neutrinos, one finds for $v_R \gg v_u$ the light neutrino mass matrix

$$m_\nu^I = -\frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T. \quad (2.5)$$

The masses of light neutrinos can also be generated through their coupling with an $SU(2)_L$ -triplet Higgs, which gives the triplet, or type II seesaw [2]:

$$m_\nu^{\text{II}} = v_L f_L. \quad (2.6)$$

In this case the neutrino mass matrix is directly given by the triplet Yukawa coupling matrix f_L , up to an overall scale which is just the triplet VEV v_L . In left-right symmetric models and their GUT extensions both seesaw contributions to m_ν are naturally present, leading to the type I + II seesaw expression of eq. (1.2). Moreover, in these models there is a relation between the VEVs of the neutral components of the two triplets v_L and v_R :

$$v_L v_R = \gamma v_u^2, \quad (2.7)$$

where γ depends on the parameters of the Higgs potential. Type I + II seesaw can, of course, also be realized without extending the gauge group of the standard model.

In general, the matrices m_ν , f_L and f_R are complex symmetric, whereas Y_D is a general complex matrix (of dimension 3×3 for three generations of light and heavy neutrinos). In left-right symmetric models and their extensions, in addition to the gauge symmetry, a discrete left-right symmetry is often assumed, which can be realized either as C -conjugation or as a parity symmetry. This leads to additional constraints on the entries of the seesaw relation (1.2). Namely, in the case of C -conjugation symmetry, one has $f_L = f_R$, $Y_D = Y_D^T$, while for parity symmetry $f_L = f_R^*$, $Y_D = Y_D^\dagger$. In both cases the seesaw exhibits a curious duality property [21] (see also [22, 23]). In our study, however, we will not assume any additional constraints on the entries of eq. (1.2). As the neutrino mass matrix given by this formula contains two terms, it leads to a number of interesting possibilities for explaining the features of neutrino mixing [24].

Let us now briefly summarize the LFV formulae relevant to our discussion. In supersymmetric scenarios LFV is triggered by off-diagonal entries in the slepton mass matrix \tilde{m}_L^2 . The branching ratios for radiative decays of the charged leptons $\ell_i = e, \mu, \tau$ are

$$\text{BR}(\ell_i \rightarrow \ell_j \gamma) = \text{BR}(\ell_i \rightarrow \ell_j \nu \bar{\nu}) \frac{\alpha^3}{G_F^2 m_S^8} \left| (\tilde{m}_L^2)_{ij} \right|^2 \tan^2 \beta, \quad (2.8)$$

where m_S is a typical mass scale of SUSY particles. The values of the branching ratios $\text{BR}(\ell_i \rightarrow \ell_j \nu \bar{\nu})$ are $\text{BR}(\mu \rightarrow e \nu \bar{\nu}) \simeq 1$, $\text{BR}(\tau \rightarrow \mu \nu \bar{\nu}) \simeq 0.174$ and $\text{BR}(\tau \rightarrow e \nu \bar{\nu}) \simeq 0.178$ [25]. Current limits on the branching ratios for $\ell_i \rightarrow \ell_j \gamma$ are $\text{BR}(\mu \rightarrow e \gamma) \leq 1.2 \cdot 10^{-11}$ [26], $\text{BR}(\tau \rightarrow e \gamma) \leq 1.1 \cdot 10^{-7}$ [27] and $\text{BR}(\tau \rightarrow \mu \gamma) \leq 6.8 \cdot 10^{-8}$ [28]. One expects to improve these bounds by two to three orders of magnitude for $\text{BR}(\mu \rightarrow e \gamma)$ [29] and by one to two orders of magnitude for the other branching ratios [30].

To satisfy the requirement that the LFV branching ratios $\text{BR}(\ell_i \rightarrow \ell_j \gamma)$ be below their experimental upper bounds, one typically assumes that \tilde{m}_L^2 and all other slepton mass

and trilinear coupling matrices are diagonal at the scale M_X . Such a situation occurs for instance in mSUGRA scenarios. Off-diagonal terms get induced at low energy scales radiatively, which explains their smallness. In this case a very good approximation for the typical SUSY mass appearing in eq. (2.8) is $m_S^8 = 0.5 m_0^2 m_{1/2}^2 (m_0^2 + 0.6 m_{1/2}^2)^2$ [19], where m_0 is the universal scalar mass and $m_{1/2}$ is the universal gaugino mass at M_X . In a supersymmetric seesaw framework the radiative entries giving rise to LFV depend on the same parameters as the neutrino masses. If there is only the type I seesaw term in m_ν , the well-known result is [13]

$$(\tilde{m}_L^2)^I_{ij} = -\frac{(3m_0^2 + A_0^2)}{8\pi^2} (Y_D L Y_D^\dagger)_{ij}, \quad \text{where } L_{ij} = \delta_{ij} \ln \frac{M_X}{M_i}. \quad (2.9)$$

In the case when only the triplet term m_ν^{II} contributes to m_ν , one finds [14]

$$(\tilde{m}_L^2)^{\text{II}}_{ij} = -3 \frac{(3m_0^2 + A_0^2)}{8\pi^2} (f_L f_L^\dagger)_{ij} \ln \frac{M_X}{M_\Delta}. \quad (2.10)$$

Here and in eq. (2.9) A_0 is the universal trilinear coupling. When both terms in the mass matrix eq. (1.2) are present, their contributions to $(\tilde{m}_L^2)_{ij}$ sum up:

$$\begin{aligned} (\tilde{m}_L^2)_{ij} &= (\tilde{m}_L^2)^I_{ij} + (\tilde{m}_L^2)^{\text{II}}_{ij} \\ &= -\frac{(3m_0^2 + A_0^2)}{8\pi^2} \left[(Y_D)_{ik} (Y_D^\dagger)_{kj} \ln \frac{M_X}{M_k} + 3 (f_L f_L^\dagger)_{ij} \ln \frac{M_X}{M_\Delta} \right]. \end{aligned} \quad (2.11)$$

As the LFV branching ratios depend on the absolute value squared of this quantity, there will be a interference term between the contributions from the triplet term and from the type I seesaw term if both of them have off-diagonal entries. We will now compare the structures of two expressions:

$$m_\nu = v_L f_L - \frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T \quad \text{versus} \quad (\tilde{m}_L^2)_{ij} \propto (f_L f_L^\dagger)_{ij} + (Y_D Y_D^\dagger)_{ij}, \quad (2.12)$$

where we have omitted logarithmic corrections to $(\tilde{m}_L^2)_{ij}$. There are several possibilities, depending on the relative magnitudes of the two contributions to $(\tilde{m}_L^2)_{ij}$ and m_ν :

- (i) in the neutrino mass matrix the type I seesaw term m_ν^I dominates, and in the off-diagonal entries of the RG-induced slepton mass matrix $(\tilde{m}_L^2)^I_{ij}$ dominates. This situation is the one best studied in the literature ([17–19], for a recent review see [31]). We have nothing new to add in this case;
- (ii) in the neutrino mass matrix the triplet term m_ν^{II} dominates, and in the off-diagonal entries of the RG-induced slepton mass matrix $(\tilde{m}_L^2)^{\text{II}}_{ij}$ dominates. This situation has also been studied [15, 16], though less often than (i);
- (iii) in the neutrino mass matrix the triplet term m_ν^{II} dominates, while in the off-diagonal entries of the RG-induced slepton mass matrix $(\tilde{m}_L^2)^I_{ij}$ dominates. This situation, to our knowledge, has not been studied yet. However, there are hardly any useful statements to be made, as there is no link between neutrino masses and LFV, even if f_L and f_R are related by $f_L = f_R$ or $f_L = f_R^*$;

- (iv) in the neutrino mass matrix the conventional seesaw term m_ν^I dominates, whereas in the off-diagonal entries of the RG-induced slepton mass matrix $(\tilde{m}_L^2)_{ij}^{II}$ dominates. Again, this situation remains to be investigated. However, as in case (iii), there is hardly any link between neutrino masses and LFV, even if f_L and f_R are related;
- (v) both terms are of comparable magnitude both in m_ν and in the off-diagonal entries of the slepton mass matrix. This case will be of prime interest to us.

In the next section we will propose a Yukawa coupling parameterization to deal with case (v), which in principle can also be applied to cases (iii) and (iv).

Before we turn to the Yukawa coupling parameterization, let us summarize the relevant leptogenesis formulae. We will assume, as it has been done in most studies, that the heavy Majorana neutrinos are lighter than the Higgs triplets. In that case it is sufficient to consider only the decay of heavy neutrinos into lepton and Higgs doublets (and similarly for the SUSY partners), while the decays of the triplets into two lepton doublets can be neglected. The CP-violating decay asymmetries of heavy neutrinos N_i contain two contributions. The first one is the same as in the case of pure type I seesaw [7, 5]:

$$\begin{aligned}
 (\varepsilon_i^\alpha)_N &= \frac{1}{8\pi} \frac{1}{(Y_D^\dagger Y_D)_{ii}} \sum_{j \neq i} \text{Im} \left[(Y_D^\dagger)_{i\alpha} (Y_D)_{\alpha j} (Y_D^\dagger Y_D)_{ij} \right] f(M_j^2/M_i^2) \\
 &\quad + \frac{1}{8\pi} \frac{1}{(Y_D^\dagger Y_D)_{ii}} \sum_{j \neq i} \text{Im} \left[(Y_D^\dagger)_{i\alpha} (Y_D)_{\alpha j} (Y_D^\dagger Y_D)_{ji} \right] \frac{1}{1 - M_j^2/M_i^2},
 \end{aligned}
 \tag{2.13}$$

where

$$f(x) = \sqrt{x} \left[\frac{2}{1-x} - \ln \left(\frac{1+x}{x} \right) \right].
 \tag{2.14}$$

We have indicated here that flavor effects [32] might play a role, i.e., ε_i^α describes the decay of the heavy neutrino of mass M_i into leptons of flavor $\alpha = e, \mu, \tau$. We will focus here on the case when the lowest-mass heavy neutrino is much lighter than the other two, i.e. $M_1 \ll M_{2,3}$; the lepton asymmetry is then dominated by the decay of this lightest neutrino. In this case $f(M_j^2/M_1^2) \simeq -3 M_1/M_j$, and in addition the second term in eq. (2.13) is strongly suppressed, therefore we will neglect it in what follows.

The second type of asymmetry is encountered when a Higgs triplet is exchanged in the loop diagrams [9]:

$$(\varepsilon_i^\alpha)_\Delta = \frac{3}{8\pi} \frac{M_i v_L}{v_u^2} \frac{1}{(Y_D^\dagger Y_D)_{ii}} \text{Im} \left[(f_L^* Y_D)_{\alpha i} (Y_D^T)_{i\alpha} \right] g(M_\Delta^2/M_i^2),
 \tag{2.15}$$

where

$$g(x) = x \ln \left(\frac{1+x}{x} \right).
 \tag{2.16}$$

In the limit $M_\Delta \gg M_{1,2,3}$, which we will assume, one has $g(x) \simeq 1 - \frac{1}{2x} = 1 - \frac{1}{2} (M_i/M_\Delta)^2$. The total asymmetries $(\varepsilon_i)_N$ and $(\varepsilon_i)_\Delta$ are obtained by summing $(\varepsilon_i^\alpha)_N$ and $(\varepsilon_i^\alpha)_\Delta$ over the flavor index α .

The baryon asymmetry of the universe ($\eta_B = n_B/n_\gamma = 6.1 \cdot 10^{-10}$) is finally found as

$$\eta_B \simeq -0.96 \cdot 10^{-2} \sum_\alpha \varepsilon_1^\alpha \kappa^\alpha,
 \tag{2.17}$$

where the washout factors κ^α are obtained by solving the relevant Boltzmann equations. The approximate expression we use is [33, 34]

$$\kappa^\alpha \simeq \frac{2}{K^\alpha z_B(K^\alpha)} \left\{ 1 - \exp \left[-K^\alpha z_B(K^\alpha)/2 \right] \right\}, \quad (2.18)$$

where $K^\alpha = \sum K_i^\alpha$ with $K_i^\alpha = |(Y_D)_{\alpha i}|^2 K_i / (Y_D^\dagger Y_D)_{ii}$ and [33]

$$z_B(K^\alpha) = 2 + 4 (K^\alpha)^{0.13} \exp(-2.5/K^\alpha). \quad (2.19)$$

Note that this formula is valid for an initial thermal abundance of the heavy neutrinos, an assumption we make for simplicity. The parameter K_i in the expression for K_i^α is defined as $K_i = \Gamma_i / H(T)|_{T=M_i}$, with the tree-level decay width of the i th heavy neutrino $\Gamma_i = (Y_D^\dagger Y_D)_{ii} M_i / (8\pi)$ and the Hubble parameter $H(T) = 1.66 \sqrt{g^*} T^2 / M_{\text{Pl}}$. The above approximate expression for $z_B(K^\alpha)$ has been obtained in ref. [33] by extending the analytic procedure from ref. [34] from the one-flavor case to the flavored case. In the examples presented in section 3 we take into account the uncertainty related to flavor effects and the approximate nature of the analytic expression for the washout factors by requiring the baryon asymmetry to lie in the interval $\eta_B = (5.7 \div 6.6) \cdot 10^{-10}$. This has to be compared with the measured value $\eta_B = (6.0 \div 6.3) \cdot 10^{-10}$ (at 68 % C.L.) from the WMAP 5 year data [35].

3. Dirac-type Yukawa coupling parameterization for type I + II seesaw and its applications

When the triplet term m_ν^{II} is present in the seesaw relation, the procedure that led to the Casas-Ibarra parameterization (1.3) of the matrix Y_D cannot be directly applied. However, as we shall show, a simple transformation of eq. (1.2) makes it possible to generalize the parameterization (1.3) to the case of type I + II seesaw.

First, we move the type II contribution to the left hand side of eq. (1.2), which gives

$$m_\nu - v_L f_L = -\frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T. \quad (3.1)$$

It is convenient to introduce the notation

$$X_\nu \equiv m_\nu - v_L f_L, \quad \text{diagonalized as} \quad X_\nu = V_\nu^* X_\nu^{\text{diag}} V_\nu^\dagger \quad (3.2)$$

with a unitary matrix V_ν . Multiplying both sides of eq. (3.1) by $X_\nu^{-1/2}$, we find

$$\mathbb{1} = -\frac{v_u^2}{v_R} (X_\nu^{-1/2} Y_D f_R^{-1/2}) (f_R^{-1/2} Y_D^T X_\nu^{-1/2}). \quad (3.3)$$

In what follows, we will be assuming that all eigenvalues of f_R (as well as those of X_ν) are distinct.³ It can be shown that in this case the matrices $X_\nu^{1/2}$ and $f_R^{1/2}$ and their respective

³In the examples considered in the next subsections X_ν has two zero eigenvalues, which should be understood as a limiting case of small distinct non-zero eigenvalues. In other words, the expressions for X_ν used there, which lead to $X_\nu^{\text{diag}} = \text{diag}(0, 0, a)$, should be understood as approximations to the exact X_ν , for which $X_\nu^{\text{diag}} = \text{diag}(\varepsilon_1, \varepsilon_2, a)$ with $|\varepsilon_1|, |\varepsilon_2| \ll |a|$.

inverse matrices are symmetric, and therefore one can rewrite eq. (3.3) as

$$\mathbb{1} = RR^T \quad \text{with} \quad R = \pm i \frac{v_u}{\sqrt{v_R}} (X_\nu^{-1/2} Y_D f_R^{-1/2}). \quad (3.4)$$

Eq. (3.4) means that the type I + II seesaw relation requires R to be an (in general complex) orthogonal matrix, but otherwise does not constrain it. Thus, for the Dirac-type Yukawa coupling Y_D we have

$$v_u Y_D = \pm i \sqrt{v_R} X_\nu^{1/2} R f_R^{1/2}, \quad (3.5)$$

where R is an arbitrary complex orthogonal matrix. It can be parameterized as

$$R = \text{diag}(\pm 1, \pm 1, \pm 1) R_{12} R_{13} R_{23}, \quad (3.6)$$

where R_{ij} is the matrix of rotation by a complex angle $\omega_{ij} = \rho_{ij} + i \sigma_{ij}$ in the ij -plane. The parameterization of the Yukawa coupling matrix Y_D in eq. (3.5) with R given by eq. (3.6) is the most general one satisfying the combined type I + II seesaw formula. Note that it is sufficient to consider the reflections described by the matrix $\text{diag}(\pm 1, \pm 1, \pm 1)$ in eq. (3.6) with only one minus sign. Indeed, a reflection with an even number of minus signs is equivalent to a rotation and can be absorbed into a redefinition of the complex angles ω_{ij} , whereas the reflection with three minus signs amounts to flipping the overall sign of the matrix Y_D . Such a sign change is irrelevant, because Y_D can in any case be reconstructed from the seesaw relation only up to the sign (see eq. (3.5)). In our examples considered below, all the reflections either leave the results unchanged or lead to the overall sign flip of Y_D ; for this reason in subsections 3.1, 3.2 and 3.3 we restrict ourselves to the case $R = R_{12} R_{13} R_{23}$.

As was pointed out above, when the underlying theory possesses a discrete left-right symmetry, type I + II seesaw exhibits a duality property [21]. In that case the seesaw relation (1.2) is invariant with respect to the duality transformation $f_R \rightarrow \hat{f}_R \equiv m_\nu/v_L - f_L$. It is interesting to note that in terms of f_R and its dual \hat{f}_R eq. (3.5) can be rewritten as

$$v_u Y_D = \pm i \sqrt{v_L v_R} \hat{f}_R^{1/2} R f_R^{1/2}. \quad (3.7)$$

It is worth stressing the difference between the approach to type I + II seesaw adopted in refs. [21–23] and the one presented here. In refs. [21–23] Y_D and m_ν were assumed to be known, and the seesaw relation was solved for the matrix $f_R = f_L$ (or $f_R = f_L^*$); eight solutions were found, forming four dual pairs. In our case we assume m_ν , $m_\nu^{\text{II}} = f_L v_L$ and $M_R = f_R v_R$ (or equivalently m_ν^{I} , m_ν^{II} and M_R) to be known and reconstruct Y_D . We find infinitely many solutions, parameterized in terms of an arbitrary complex orthogonal matrix R .

For practical applications, it proves to be convenient to use a slightly modified version of eq. (3.5). First, we note that for discussions of both LFV and leptogenesis one has to go to the basis where the mass matrix of heavy Majorana neutrinos M_R is diagonal and real. As $M_R = f_R v_R$, this also diagonalizes the matrix f_R . The corresponding transformation is

$$V_R^T f_R V_R = f_R^{\text{diag}}, \quad (3.8)$$

with a unitary matrix V_R . Note that $f_R^{\text{diag}} = \text{diag}(M_1, M_2, M_3)/v_R$. The transformation (3.8) amounts to replacing the Yukawa coupling matrix Y_D in the seesaw relation according to $Y_D \rightarrow Y_D V_R$, i.e. it fixes its right-handed basis. In what follows we will be assuming that the matrix M_R has been diagonalized and consider Y_D in this basis, except in example 3 below, where this diagonalization will be carried out explicitly. Next, it is convenient to express X_ν through its eigenvalues. To this end, using eq. (3.2) we rewrite X_ν on the left hand side of eq. (3.1) as

$$X_\nu = V_\nu^* X_\nu^{\text{diag}} V_\nu^\dagger = [V_\nu^* (X_\nu^{\text{diag}})^{1/2}] [V_\nu^* (X_\nu^{\text{diag}})^{1/2}]^T. \quad (3.9)$$

Multiplying then eq. (3.1) by $[V_\nu^* (X_\nu^{\text{diag}})^{1/2}]^{-1}$ on the left and by $\{[V_\nu^* (X_\nu^{\text{diag}})^{1/2}]^T\}^{-1}$ on the right and following the same steps as above, one readily finds

$$v_u Y_D = \pm i \sqrt{v_R} V_\nu^* \sqrt{X_\nu^{\text{diag}}} R \sqrt{f_R^{\text{diag}}}. \quad (3.10)$$

This parameterization is the main point of the present paper and we will be using it in the subsequent discussion. Note that the matrix R here is in general not the same as the matrix R in eq. (3.5). This is of no concern to us, as both are arbitrary complex symmetric matrices. We note that our parameterization is useful — just like the Casas-Ibarra parameterization — in the basis in which the charged lepton Yukawa matrix is real and diagonal. A remark is also in order on why we assume that the triplet contribution to m_ν is known, while Y_D is not (note that since we also assume m_ν to be known, the knowledge of m_ν^{II} implies that m_ν^{I} is known as well). In general, the matrices $m_\nu^{\text{II}} = f_L v_L$ and Y_D stem from different Higgs sectors, and therefore by fixing one of them one does not necessarily determine the other. Normally, to fix Y_D one invokes some additional assumptions, such as quark-lepton symmetry. Our parameterization allows one to find phenomenologically viable structures of Y_D and can therefore be useful for building neutrino mass models and for studying their implications for LFV and leptogenesis.

In the remainder of this section we will give simple examples demonstrating the usefulness of the parameterization (3.10). In the first two examples we consider tri-bimaximal neutrino mixing [36], which describes very well the current status of global fits to the low energy neutrino data. The neutrino mass matrix giving rise to tri-bimaximal mixing can be written as

$$m_\nu = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_2 e^{2i\alpha}}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_3 e^{2i\beta}}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}. \quad (3.11)$$

If neutrinos enjoy the normal mass hierarchy, one can neglect m_1 , so that the first term in eq. (3.11) vanishes, and in addition one has $m_2 = \sqrt{\Delta m_\odot^2}$ and $m_3 = \sqrt{\Delta m_A^2}$. An appealing possibility in this case is that the two remaining individual matrices in eq. (3.11) correspond to m_ν^{I} and m_ν^{II} , respectively [24]. The moderate ratio of the two terms in m_ν is therefore $\frac{3}{2} \sqrt{\Delta m_A^2 / \Delta m_\odot^2} \simeq 8.4$. We will investigate this possibility and apply our parameterization of Y_D to this case in the following two examples. The third example will be based on a perturbation of bimaximal leptonic mixing [37] in the type I + II seesaw framework.

3.1 First example

Suppose first that the triplet term m_ν^{II} is the term proportional to m_3 in eq. (3.11), i.e.

$$f_L = \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix} e^{2i\beta} \quad \text{and} \quad v_L = \sqrt{\Delta m_A^2}/2. \quad (3.12)$$

The second, flavor democratic term proportional to m_2 , is then provided by the conventional type I seesaw. Due to the seesaw relation (3.1) it determines X_ν :

$$X_\nu = -\frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T = \frac{m_2 e^{2i\alpha}}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}.$$

Consequently, one can write $X_\nu^{\text{diag}} = \text{diag}(0, 0, m_2)$ and

$$V_\nu = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \text{diag}(1, 1, e^{-i\alpha}). \quad (3.13)$$

The scales involved are $v_L \simeq 0.025 \text{ eV}$, $v_R = 3v_u^2/\sqrt{\Delta m_\odot^2} \simeq 1.0 \cdot 10^{16} \text{ GeV}$ (assuming $v_u = 174 \text{ GeV}$, which is an excellent approximation as long as $\tan\beta \gtrsim 5$), and $\gamma \simeq 8.4$. Note that we have rather arbitrarily decomposed the second and third terms in eq. (3.11) into the VEVs and Yukawa couplings or their combinations.

We have now all ingredients to express Y_D through eq. (3.10), and the result is

$$Y_D = -ie^{i\alpha} \frac{\sqrt{m_2}}{\sqrt{3}v_u} \begin{pmatrix} \sqrt{M_1} \sin\omega_{13} & \sqrt{M_2} \cos\omega_{13} \sin\omega_{23} & -\sqrt{M_3} \cos\omega_{13} \cos\omega_{23} \\ \sqrt{M_1} \sin\omega_{13} & \sqrt{M_2} \cos\omega_{13} \sin\omega_{23} & -\sqrt{M_3} \cos\omega_{13} \cos\omega_{23} \\ \sqrt{M_1} \sin\omega_{13} & \sqrt{M_2} \cos\omega_{13} \sin\omega_{23} & -\sqrt{M_3} \cos\omega_{13} \cos\omega_{23} \end{pmatrix}. \quad (3.14)$$

Interestingly, the complex angle ω_{12} drops out of this expression.

Let us now discuss LFV in the considered example. Eq. (3.12) yields

$$f_L f_L^\dagger = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad (3.15)$$

from which it follows that only the decay $\tau \rightarrow \mu\gamma$ is influenced by the triplet term. The decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ depend only on $Y_D Y_D^\dagger$, which has a democratic structure with all terms equal to each other. Consequently, $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ decays are governed by the same quantity:

$$\begin{aligned} |(Y_D Y_D^\dagger)_{21} + 3(f_L f_L^\dagger)_{21}|^2 &= |(Y_D Y_D^\dagger)_{31} + 3(f_L f_L^\dagger)_{31}|^2 \\ &= \frac{m_2^2}{9v_u^4} (M_1 |\sin\omega_{13}|^2 + |\cos\omega_{13}|^2 (M_2 |\sin\omega_{23}|^2 + M_3 |\cos\omega_{23}|^2))^2. \end{aligned} \quad (3.16)$$

This equality implies that $\text{BR}(\tau \rightarrow e\gamma) = 0.178 \text{BR}(\mu \rightarrow e\gamma)$. With the current limit of $1.2 \cdot 10^{-11}$ on $\text{BR}(\mu \rightarrow e\gamma)$, and an expected improvement of two orders of magnitude on the limit of $\text{BR}(\tau \rightarrow e\gamma) \leq 1.1 \cdot 10^{-7}$, it follows that in this scenario $\tau \rightarrow e\gamma$ will not be observed in a foreseeable future. The branching ratio of the decay $\tau \rightarrow \mu\gamma$ depends on

$$\left(Y_D Y_D^\dagger\right)_{32} + 3 \left(f_L f_L^\dagger\right)_{32} = \left(Y_D Y_D^\dagger\right)_{21} - 6. \quad (3.17)$$

We have omitted here the logarithmic dependence on the masses of the triplet and of the heavy Majorana neutrinos. In the plots to be shown in the following we use the full expressions, however. As follows from eqs. (3.16) and (3.17), the matrix $Y_D Y_D^\dagger + 3 f_L f_L^\dagger$ depends in general on two complex angles, ω_{23} and ω_{13} . If degenerate heavy Majorana masses are assumed, $M_1 = M_2 = M_3$, then the real part of ω_{23} drops out of this matrix.

Turning to leptogenesis, the first thing to note is that all $(\varepsilon_i^\alpha)_\Delta$ vanish, which is a consequence of the fact that the matrix $f_L^* Y_D$ vanishes identically. The decay asymmetry is therefore the same as for pure type I seesaw. The individual flavored asymmetries $(\varepsilon_1^\alpha)_N$ are all identical and equal to one third of the total asymmetry. For hierarchical heavy neutrinos we find

$$\begin{aligned} (\varepsilon_1^e)_N = (\varepsilon_1^\mu)_N = (\varepsilon_1^\tau)_N &= \frac{1}{3} \varepsilon_1^N \simeq \frac{1}{16\pi} \frac{m_2 M_1}{v_u^2} \frac{\sin 2\rho_{13} \sinh 2\sigma_{13}}{|\sin \omega_{13}|^2} \\ &\simeq 6 \cdot 10^{-9} \left(\frac{M_1}{10^9 \text{ GeV}} \right) \frac{\sin 2\rho_{13} \sinh 2\sigma_{13}}{|\sin \omega_{13}|^2}. \end{aligned} \quad (3.18)$$

Hence, only the complex angle ω_{13} plays a role here. Terms containing ω_{23} appear in the decay asymmetry multiplied by $f(M_2^2/M_1^2)M_2 - f(M_3^2/M_1^2)M_3$, which vanishes in the limit of hierarchical heavy neutrinos. If ω_{13} is zero, then N_1 decouples (see eq. (3.14)), and N_2 will be responsible for leptogenesis. The low energy (Majorana) phases α and β do not contribute to either ε_1 or to ε_2 , i.e. play no role in leptogenesis.

Choosing $M_X = 2 \cdot 10^{16}$ GeV, $M_\Delta = 5 \cdot 10^{15}$ GeV and the masses of heavy Majorana neutrinos $M_1 = 10^{10}$ GeV, $M_2 = 10^{12}$ GeV, and $M_3 = 10^{15}$ GeV, we show in figure 1 the baryon asymmetry against the imaginary part of ω_{13} . All free parameters were varied, the baryon asymmetry was required to be positive and the branching ratios of $\mu \rightarrow e\gamma$ (which in the considered example coincides with $\text{BR}(\tau \rightarrow e\gamma)/0.178$) and of $\tau \rightarrow \mu\gamma$ were required to lie below their current upper limits. The supersymmetric parameters we have used correspond to the SPS benchmark point 2 of ref. [38] and are $\tan \beta = 10$, $m_0 = 1450$ GeV, $m_{1/2} = 300$ GeV and $A_0 = 0$.⁴ The apparent symmetry of figure 1 around the value $\text{Im}(\omega_{13}) = \sigma_{13} = 0$ can be explained by the dependence of the decay asymmetry (3.18) on ω_{13} . For all other parameters fixed, changing the sign of σ_{13} would also change the sign of the decay asymmetry. To regain the correct sign of the baryon asymmetry one would then also have to flip the sign of $\sin 2\rho_{13}$ (recall that ρ_{13} is varied as a free parameter in this scatter plot), leading to the apparent symmetry of the figure.

⁴For completeness, we quote here the outcome for the other SPS benchmark points: the results for the LFV branching ratios would be about 24 times larger for SPS 1a and 1b, and roughly 200 times larger for SPS 4. The results for SPS 3 are about 7% larger than those we quote for SPS 2.

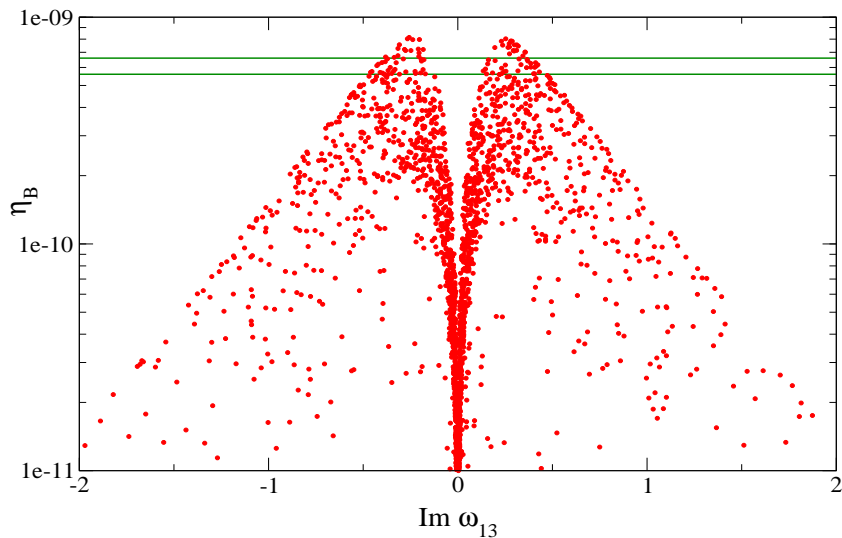


Figure 1: Scatter plot for the baryon asymmetry η_B against the imaginary part of ω_{13} for the first example of section 3.1. The observed value of η_B corresponds to the region between the horizontal lines.

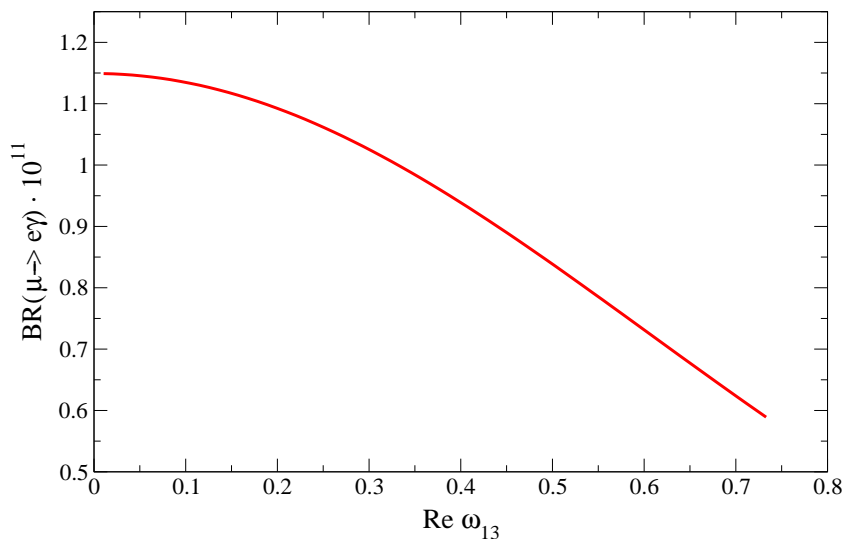


Figure 2: The branching ratio of $\mu \rightarrow e\gamma$ decay against the real part of ω_{13} for a particular point in the parameter space of the first example of section 3.1 (see the text for details).

The branching ratio $\text{BR}(\tau \rightarrow \mu\gamma)$ is basically independent of the parameters of Y_D , because the constant term in eq. (3.17) turns out to be much larger than the Y_D -dependent one. The ratio $\text{BR}(\mu \rightarrow e\gamma)/\text{BR}(\tau \rightarrow \mu\gamma)$ is of order 10^{-4} , implying that $\tau \rightarrow \mu\gamma$ is observable as long as $\text{BR}(\mu \rightarrow e\gamma)$ is close to its current limit. Fixing in addition $\rho_{23} = 1.7$, $\sigma_{23} = -0.3$ and $\sigma_{13} = -0.7$, we show in figure 2 the branching ratio of $\mu \rightarrow e\gamma$ against the remaining free parameter $\rho_{13} = \text{Re}(\omega_{13})$. For this particular point $\text{BR}(\tau \rightarrow \mu\gamma) \simeq$

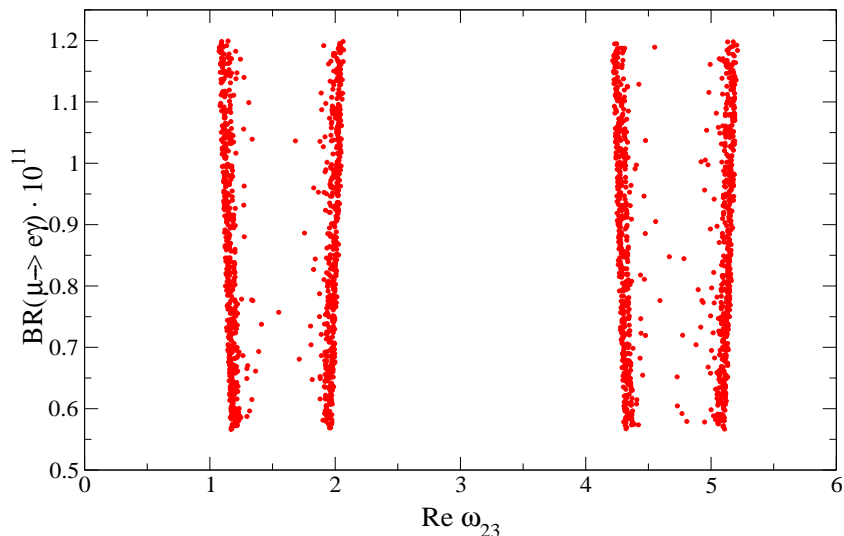


Figure 3: First example from section 3.1: scatter plot for the branching ratio of $\mu \rightarrow e\gamma$ decay against the real part of ω_{23} when the baryon asymmetry η_B is within its experimental range.

$5.05 \cdot 10^{-8}$, which is very close to its current upper limit. Figure 3 shows a scatter plot for the branching ratio of $\mu \rightarrow e\gamma$ against the real part of ω_{23} when the baryon asymmetry is within its allowed range. The symmetry around the value $\text{Re}(\omega_{23}) = \rho_{23} = \pi$ of this plot can be understood by noting that the term proportional to $M_3 |\cos \omega_{23}|^2$ is the leading one in eq. (3.16). This term depends on ρ_{23} through $\cos 2\rho_{23}$.

3.2 Second example

Let us now consider the situation in which the triplet term is flavor democratic, i.e.,

$$f_L = \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} e^{2i\alpha} \quad \text{and} \quad v_L = \sqrt{\Delta m_\odot^2}/3.$$

The remaining term in eq. (3.11) is then

$$X_\nu = -\frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T = \frac{m_3 e^{2i\beta}}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}.$$

The involved scales are $v_L \simeq 0.003 \text{ eV}$, $v_R = 2 v_u^2 / \sqrt{\Delta m_A^2} \simeq 1.2 \cdot 10^{15} \text{ GeV}$, and $\gamma \simeq 0.12$. Here we have taken $v_u^2/v_R = m_3/2 = \sqrt{\Delta m_A^2}/2$. The matrix X_ν is diagonalized by

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \text{diag}(1, 1, e^{-i\beta}). \tag{3.19}$$

with $X_\nu^{\text{diag}} = \text{diag}(0, 0, m_3)$. From eq. (3.10) we then obtain

$$Y_D = -i e^{i\beta} \frac{\sqrt{m_3}}{\sqrt{2} v_u} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{M_1} \sin \omega_{13} & \sqrt{M_2} \cos \omega_{13} \sin \omega_{23} & -\sqrt{M_3} \cos \omega_{13} \cos \omega_{23} \\ -\sqrt{M_1} \sin \omega_{13} & -\sqrt{M_2} \cos \omega_{13} \sin \omega_{23} & \sqrt{M_3} \cos \omega_{13} \cos \omega_{23} \end{pmatrix}. \quad (3.20)$$

Note that, as in the previous example, Y_D does not depend on ω_{12} . The matrix $Y_D Y_D^\dagger$ has zero first row and first column, therefore $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ decays are governed by the triplet contribution, and depend on the the same quantity, namely $(f_L f_L^\dagger)_{21} = (f_L f_L^\dagger)_{31} = 3$ (note that $f_L f_L^\dagger = 3 f_L e^{-2i\alpha}$, i.e. is flavor democratic and has no dependence on any of the free parameters). Consequently, the decay $\tau \rightarrow e\gamma$ will not be observed in a near future. The fact that $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ decays depend on the same quantity in both our examples is a consequence of the μ - τ symmetry of the involved mass matrices. Finally,

$$\begin{aligned} & (Y_D Y_D^\dagger)_{32} + 3 (f_L f_L^\dagger)_{32} \\ &= \frac{1}{2v_u^2} |18 v_u^2 - m_3 (M_1 |\sin \omega_{13}|^2 + |\cos \omega_{13}|^2 (M_2 |\sin \omega_{23}|^2 + M_3 |\cos \omega_{23}|^2))|. \end{aligned} \quad (3.21)$$

Because the 12- and 13-entries of $f_L f_L^\dagger$ are independent of any free parameters, it is not possible to suppress them, and in general the branching ratios of LFV decays are too large unless the SUSY masses are around or above 10 TeV.

Let us now turn to leptogenesis. As in the previous example, all $(\varepsilon_i^\alpha)_\Delta$ vanish because the matrix $f_L^* Y_D$ vanishes identically. The decay asymmetry $(\varepsilon_1^e)_N$ is also zero, whereas $(\varepsilon_1^\mu)_N$ and $(\varepsilon_1^\tau)_N$ are identical, and equal to $\frac{1}{2} (\varepsilon_1)_N$. In the limit of the hierarchical heavy neutrino masses we find

$$\begin{aligned} (\varepsilon_1^\mu)_N = (\varepsilon_1^\tau)_N &= \frac{1}{2} (\varepsilon_1)_N = \frac{3}{32\pi} \frac{m_3 M_1}{v_u^2} \frac{\sin 2\rho_{13} \sinh 2\sigma_{13}}{|\sin \omega_{13}|^2} \\ &\simeq 5 \cdot 10^{-8} \left(\frac{M_1}{10^9 \text{ GeV}} \right) \frac{\sin 2\rho_{13} \sinh 2\sigma_{13}}{|\sin \omega_{13}|^2}. \end{aligned} \quad (3.22)$$

The dependence of these asymmetries on the complex angle ω_{13} is identical to that in the first example considered above. Note that here the decay asymmetry is proportional to the mass of the heaviest of light neutrinos m_3 , whereas it was proportional to m_2 in the first example.

3.3 Third example

Our final example is based on the following observation [39, 24]: if the triplet term corresponds to bimaximal mixing [37] ($U_{e3} = 0$ and $\theta_{12} = \theta_{23} = \pi/4$), then a small contribution from the conventional type I seesaw term may shift θ_{12} sufficiently away from the maximal mixing value to make it agree with data. Non-zero θ_{13} and non-maximal mixing in the 2-3 sector are also generated. It was assumed in [39, 24] that m_D is hierarchical, symmetric and coincides with the up-type quark mass matrix. The triplet term $v_L f_L$ alone would

generate bimaximal neutrino mixing and a normal mass ordering with a non-vanishing smallest neutrino mass. A discrete left-right symmetry is also assumed, such that $f_L = f_R$. It is easy to see that in this case the type I seesaw term contributes to m_ν mainly a 33 entry $v_L \eta$, which is suppressed with respect to the leading (order v_L) term of m_ν^{II} [11]. The other elements of m_ν^{I} are much smaller than $v_L \eta$, and we will neglect them. It should be noted that many other Dirac mass matrices can also give the desired form $m_\nu^{\text{I}} \propto \text{diag}(0, 0, 1)$, and our parameterization allows to study them all.

The triplet contribution is

$$f_L = f_R = \begin{pmatrix} \epsilon & B\epsilon & B\epsilon \\ \cdot & \frac{1}{2}(\epsilon + e^{i\phi}) & \frac{1}{2}(\epsilon - e^{i\phi}) \\ \cdot & \cdot & \frac{1}{2}(\epsilon + e^{i\phi}) \end{pmatrix} \quad \text{and} \quad v_L = \frac{\sqrt{\Delta m_A^2}}{2}.$$

For simplicity we assume the order one parameter B and $\epsilon \ll 1$ to be real. The product $v_L \epsilon$ is of the order of $\sqrt{\Delta m_\odot^2}$.

The type I contribution we require is

$$X_\nu = -\frac{v_u^2}{v_R} Y_D f_R^{-1} Y_D^T \equiv v_L \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 0 & 0 \\ \cdot & \cdot & \eta \end{pmatrix}.$$

The involved scales are $v_L = 0.025 \text{ eV}$ and $v_R = 2v_u^2/\sqrt{\Delta m_A^2} \simeq 1.2 \cdot 10^{15} \text{ GeV}$. As a consequence of non-zero η , the zeroth-order values $U_{e3} = 0$ and $\theta_{12} = \theta_{23} = \pi/4$ are modified to $|U_{e3}| \simeq B\epsilon\eta/\sqrt{2}$, $\tan^2 \theta_{23} \simeq 1 - 2\eta$ and $\tan 2\theta_{12} \simeq 4\sqrt{2}B\epsilon/\eta$, where for simplicity also η is assumed to be real. The value $\sin^2 \theta_{12} = \frac{1}{3}$ is achieved for $B\epsilon = \eta/2$. The ratio of the neutrino mass squared differences $\Delta m_\odot^2/\Delta m_A^2$ is approximately $\frac{3}{4}\eta(4\epsilon + \eta)$. A choice of parameters which leads to neutrino properties that agree with the data, and which we will use in what follows, is $B = 1.1$, $\eta = 0.1194$ and $\epsilon = 0.0542$. The low energy phase ϕ is the Dirac-type CP violation phase which can influence neutrino oscillations.

Since X_ν is diagonal, we have $V_\nu = \mathbb{1}$, whereas the matrix diagonalizing f_R via $V_R^T f_R V_R = f_R^{\text{diag}}$ is

$$V_R = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\sqrt{\frac{1}{2}} \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix} P_R,$$

with $P_R = \text{diag}(i, 1, e^{-i\phi/2})$. The eigenvalues of f_R are $\epsilon(1 - \sqrt{2}B)$, $\epsilon(1 + \sqrt{2}B)$ and $e^{i\phi}$. Since we started in a basis in which $f_R = f_L$ is not diagonal, we have to use a modified parameterization for Y_D , which is obtained from eq. (3.10) by multiplying it on the right

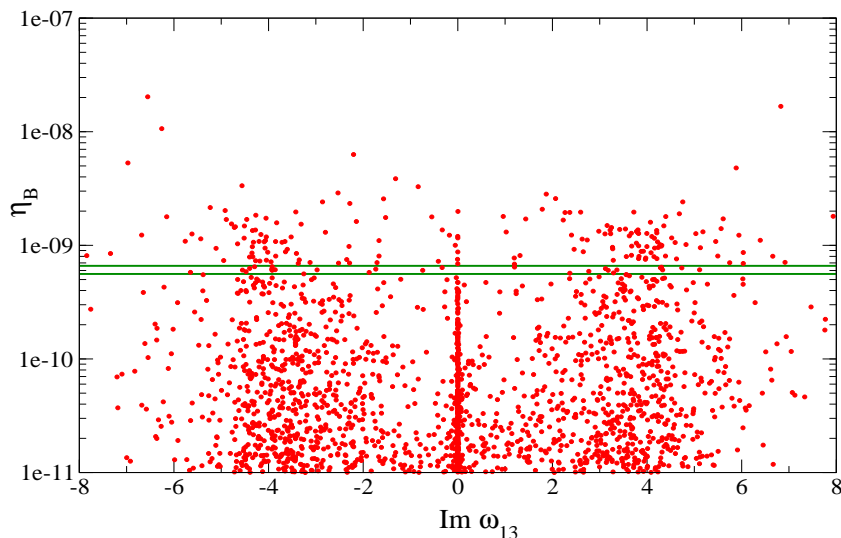


Figure 4: Scatter plot for the baryon asymmetry η_B against the imaginary part of ω_{13} for the third example of section 3.3. The observed value of η_B corresponds to the region between the horizontal lines.

by V_R . The Dirac-type Yukawa coupling matrix is then found to be

$$\begin{aligned}
 Y_D &= i(\sqrt{v_R}/v_u) V_\nu^* \sqrt{X_\nu^{\text{diag}}} R \sqrt{f_R^{\text{diag}}} V_R \\
 &= i(\sqrt{v_R v_L}/v_u) \text{diag}(0, 0, \sqrt{\eta}) R \sqrt{f_R^{\text{diag}}} V_R, \\
 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (Y_D)_{31} & (Y_D)_{32} & (Y_D)_{33} \end{pmatrix}
 \end{aligned} \tag{3.23}$$

where the non-zero entries are

$$\begin{aligned}
 (Y_D)_{31} &= \frac{\sqrt{v_L \eta}}{2 v_u} \left(\sqrt{2} \sqrt{M_1} \sin \omega_{13} + \cos \omega_{13} (\sqrt{M_3} \cos \omega_{23} - \sqrt{M_2} \sin \omega_{23}) \right), \\
 (Y_D)_{32} &= -i \frac{\sqrt{v_L \eta}}{2 v_u} \left(\sqrt{2} \sqrt{M_1} \sin \omega_{13} - \cos \omega_{13} (\sqrt{M_3} \cos \omega_{23} - \sqrt{M_2} \sin \omega_{23}) \right), \\
 (Y_D)_{33} &= i \frac{\sqrt{v_L \eta}}{\sqrt{2} v_u} e^{-i \phi/2} \cos \omega_{13} (\sqrt{M_3} \cos \omega_{23} + \sqrt{M_2} \sin \omega_{23}).
 \end{aligned}$$

Note that in all three examples we have considered so far, Y_D does not depend on ω_{12} . This is related to the fact that in all these examples the matrix X_ν^{diag} has only one (namely, third) non-vanishing diagonal entry.

The result for LFV in the present example is that the branching ratios of the decays $\ell_i \rightarrow \ell_j \gamma$ depend only on $f_L f_L^\dagger$, namely, $(f_L f_L^\dagger)_{12} = (f_L f_L^\dagger)_{13} = 2 \epsilon^2 B$ and $(f_L f_L^\dagger)_{23} = -\frac{1}{2}[1 - \epsilon^2 (1 + 2 B^2)]$. As in the previous two examples, $\tau \rightarrow e \gamma$ is too rare to be observable. The ratio $\text{BR}(\mu \rightarrow e \gamma)/\text{BR}(\tau \rightarrow \mu \gamma)$ is approximately $(2 \epsilon^2 B / \frac{1}{2})^2 / 0.174 \simeq 10^{-3}$.

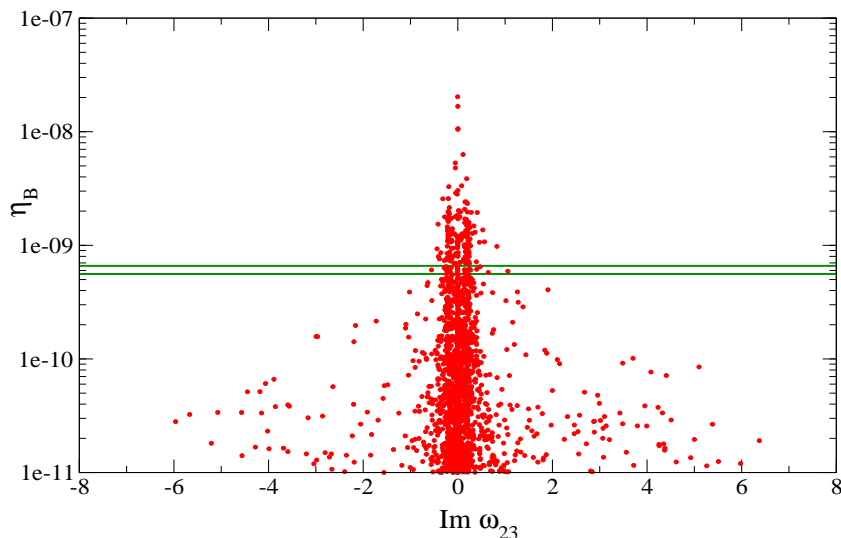


Figure 5: Scatter plot for the baryon asymmetry η_B against the imaginary part of ω_{23} for the third example of section 3.3. The observed value of η_B corresponds to the region between the horizontal lines.

Turning to leptogenesis, only $(\varepsilon_1^\tau)_N$ and $(\varepsilon_1^\tau)_\Delta$ are non-zero. The corresponding expressions are rather lengthy and we do not give them here. Figures 4 and 5 show scatter plots of the baryon asymmetry against the imaginary parts of ω_{13} and ω_{23} for fixed values of the LFV branching ratios. For definiteness, we have chosen again the SUSY parameters $\tan\beta = 10$, $m_0 = 1450$ GeV, $m_{1/2} = 300$ GeV and $A_0 = 0$, which gives $\text{BR}(\mu \rightarrow e\gamma) = 3.0 \cdot 10^{-12}$, $\text{BR}(\tau \rightarrow e\gamma) = 5.4 \cdot 10^{-13}$ and $\text{BR}(\tau \rightarrow \mu\gamma) = 3.1 \cdot 10^{-9}$.

4. Summary and conclusions

We have considered lepton flavor violation and leptogenesis in the case of type I + II seesaw, when the exchanges of both heavy Majorana neutrinos and $SU(2)_L$ -triplet Higgs bosons contribute to the mass matrix of light neutrinos. We have proposed a parameterization of the Dirac-type neutrino Yukawa coupling matrix Y_D in this framework, which generalizes the Casas-Ibarra parameterization suggested for type I seesaw. Our parameterization automatically takes into account the type I + II seesaw formula and, like the Casas-Ibarra one, involves an arbitrary complex orthogonal matrix R . This matrix depends in general on six real parameters and can be parameterized in terms of three complex angles. We have given simple examples illustrating the usefulness of the proposed parameterization. In particular, we have considered LFV decays $\ell_i \rightarrow \ell_j\gamma$ and leptogenesis in the case when the type I and type II contributions to both the light neutrino mass matrix m_ν and the slepton mass matrix \tilde{m}_L^2 governing the LFV decays are of the same order. We considered two examples leading to the tri-bimaximal leptonic mixing and an example based on a relatively small but phenomenologically viable deviation from bimaximal mixing. In all the examples we have studied we found that the matrix Y_D depends only on two out of the

three complex angles parameterizing the matrix R , which is related to the fact that the matrix $X_\nu \equiv m_\nu - f_L v_L$ had only one non-zero eigenvalue.

In each of the three examples that we considered, we have found that the decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ are governed by the same quantity, and the corresponding branching ratios are related by $\text{BR}(\tau \rightarrow e\gamma) \simeq 0.178 \text{BR}(\mu \rightarrow e\gamma)$, which is a consequence of the approximate μ - τ symmetry of the involved mass matrices.

In the first two examples based on tri-bimaximal leptonic mixing we found that leptogenesis is essentially governed by one of the three complex angles parameterizing the matrix R . This can be traced back to the facts that the masses of heavy Majorana neutrinos were assumed to be hierarchical and that the loops with the triplet exchange gave no contribution to lepton asymmetry in these examples.

To conclude, we proposed a new parameterization of the Dirac-type neutrino Yukawa coupling matrix Y_D which is the most general one satisfying the combined type I + II seesaw formula. It expresses the matrix Y_D through both low energy and high energy parameters and can be useful for studies of lepton flavor violation and leptogenesis in the type I + II seesaw framework.

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